Exponential and Logarithmic Functions

Exercise 10.4 – Question 10:

10j)

Use common logarithms to find the solutions of the following exponential equation.

$$2.67^{2x} = 28.5$$

Solution

$2.67^{2x} = 28.5$ Writing the equation

$\log_{2.67} 2.67^{2x} = \log 28.5$ Taking the logs of both sides

$2x \log 2.67 = \log 28.5$ Using: ($\log a^n = n \times \log a$)

$\frac{2 \times \log 2.67}{2 \log 2.67} = \frac{\log 28.5}{2 \log 2.67}$ Dividing both sides by (2 log 2.67)

$\frac{\cancel{2} \times \cancel{\log 2.67}}{\cancel{2}\ \cancel{\log 2.67}} = \frac{\log 28.5}{2 \log 2.67}$ Cancelling common factors from the numerator and denominator

$$x = \frac{\log 28.5}{2 \log 2.67}$$

$$x = \frac{1{\cdot}4548}{0.8530}$$

Finding the relevant values

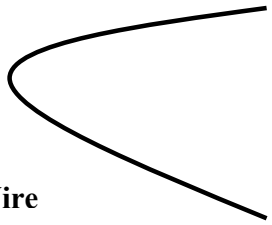
Answer: $x \approx 1.7055$.

Exercise 10.5 - Modelling and Problem Solving

Question 6:

A piece of very thin wire, 1m long, is folded in half so that its length is 50cm. Calculate the length of the bent wire after a total of 5 folds and find a rule for its length in centimeters after n folds.

Solution



The Wire

$L = a \times b^n$ Writing the general formula for the decay function

Where

L = The length of the wire after n folds
n = Number of folds
a = 100 cm (initial length)

$b = \frac{1}{2}$ The growth factor

$L = 100 \times \left(\frac{1}{2}\right)^n$ Substituting the values of a and b into the decay function

The above exponential function models the length of the wire after n folds

Calculating the length of wire after 5 folds:

Let n = 5

So:

$$L = 100 \times \left(\frac{1}{2}\right)^{5}$$

$L \approx 3.1$ cm

Answer: The length of wire after n folds is about 3.1 cm.